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On the Number of Solutions to a Crossing Symmetric Neutral π-π Model

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Abstract. The crossing symmetry of the π–π system has invited many authors to try different versions of the bootstrap hypothesis on it. In the last few years, there has been some hope that the positivity conditions of Martin, Balachandran and Nuyts, Roskies, and, most recently, the Roy physical region constraints, might be sufficient to fix the low π–π partial waves with very little additional information, like the position and width of the ϱ resonance. This hope was recently proved too optimistic by Basdevant, Froggatt and Petersen. A similar result was obtained by the present author in an approximately crossing symmetric, solvable model. In this paper we strengthen this latter result by determining the multiplicity of the exact solution to a crossing symmetric neutral π–π model. We consider only the S wave, but the multiplicity would increase by the addition of other coupled channels. The analysis is not confined to weak coupling only, and includes all solutions, in particular also a class of logarithmically decreasing ones, which are left out by most other authors.

1. Introduction

Some time ago, the present author showed in a simple solvable neutral $\pi\pi$ model [1] that the positivity conditions, derived by Martin [2] and a number of other authors, are not sufficient to fix the $\pi^0\pi^0 S$ wave when, say, the scattering length is given [3]. In fact a number of CDD poles, with arbitrary position and residue, could be added to the expression for $1/f(s)$, without violating seriously the positivity conditions.

One might argue that the model of [1] only approximately satisfies the positivity conditions, and that the result would be different in a model which satisfies the positivity conditions exactly. The obvious answer to this argument is to construct such a model, which satisfies all possible positivity conditions, and prove that its solution is not unique. This is what we shall do in the present work.

The model we shall consider is a neutral, crossing symmetric $\pi\pi$ model, which is known as the Cini-Fubini [4] approximation for the S wave. It is obtained by taking the once subtracted Mandelstam re-

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