Commun. math. Phys. 40, 83—95 (1975)

Analyticity and Uniqueness of the Invariant Equilibrium States for General Spin $\frac{1}{2}$ Classical Lattice Systems

C. Gruber, A. Hintermann, and D. Merlini

Laboratoire de Physique Théorique, Ecole Polytechnique Fédérale Lausanne, Lausanne, Switzerland

Received July 29, 1974

Abstract. Asano-Ruelle-Slawny method is generalized to discuss analyticity and uniqueness of the correlation functions in terms of the group structure associated with any lattice systems. The use of Poisson formula for abelian groups gives a simple method to obtain explicit domains where the above properties are verified.

I. Introduction

The analysis of the zeroes of the partition function Z is one of the standard methods of statistical mechanics used to derive general properties of lattice systems. A powerful technique, based on Asano's contraction [1], was given by Ruelle to study domains of zeroes of Z in the complex variable $z = e^{-2\beta h}$ [2]; the main idea is to reduce the study of the zeros of the partition function to the study of smaller polynomials associated with the *local structure of the lattice.*

More recently the Asano-Ruelle technique was generalized by Slawny [3] to discuss domains of zeroes in all the complex variables $z_B = e^{-2\beta J(B)}$ where $J(B)$ is the interaction associated with the bond B; the idea of Slawny is to start with the Low Temperature (L.T.) expansion of the partition function expressed as a summ over a certain group Γ, called the L.T. group. He then makes use of the group structure associated with lattice systems to give general conditions for the partition function to be the Asano contraction of the partition function of small subsystems. Using then a theorem due to Ruelle [4] this extension of the Asano-Ruelle technique yields new results concerning the analyticity properties of the free energy and the uniqueness of the equilibrium state for ferromagnetic systems at low temperatures.

In the following we shall extend Slawny's results to *arbitrary lattice systems* and give a general method to obtain explicit domains where Z is different from zero. These results then imply the analyticity and uniqueness properties of the free energy and the correlation functions, and they follow from a generalization of the Asano-Ruelle-Slawny method.

We consider an arbitrary spin $\frac{1}{2}$ lattice system $\{\Lambda, \mathscr{B}\}$, defined by a set of lattice sites Λ and a set $\mathscr{B}$ of bonds, $\mathscr{B} \subset \mathscr{P}(\Lambda)$; we then discuss the zeroes of polynomials of the form $M(z_{\mathscr{B}}) = \sum_{\beta \in G} \prod_{B \in \beta} z_B$ where G *is any*