Commun. math. Phys. 39, 329—344 (1975)

Current Algebra Ward Identities in the Renormalized σ Model

C. Becchi* **
Centre de Physique Théorique, C.N.R.S., Marseille, France

Received March 8, 1974

Abstract. The many-current Ward identities corresponding to the Gell-Mann current algebra are discussed in the renormalized σ model. The Ward identities are verified in the case of the $SU(2) \times SU(2)$ chiral symmetry. In the $SU(3) \times SU(3)$ case the uniqueness of the Adler-Bardeen anomaly is proved using the Wess-Zumino consistency conditions.

Introduction

In the framework of the renormalized σ model [1, 2] a multiplet of chiral currents was defined, the vector currents being conserved and the axial currents satisfying the P.C.A.C. [3] condition to any order of perturbation theory. On a heuristic basis the currents are normalized by fixing the field current commutation rules. These facts are best summarized by a system of Ward identities involving the time ordered product of one current operator and any number of quantized fields.

It remains to prove that these chiral current operators satisfy a suitable version of the commutation rules of the Gell-Mann current algebra[1]. In other words we have to extend to any order of perturbation theory the many-current Ward identities [4] which are valid in the semi-classical σ model [5] (i.e. in tree approximation). The time ordered products of any number of fields and current operators are defined [2] in the renormalized σ model up to the addition of seagull terms; that is terms which might be non zero only if the arguments of at least two currents coincide. In general the Ward identities will depend on such terms. Thus the problem is to see whether one can find a definition of the many-current T-products for which the Ward identities corresponding to the chiral current algebra are satisfied.

We shall begin studying the $SU(2) \times SU(2)$ σ model; in this case the current algebra Ward identities will be proved. Then we shall extend

* On leave of absence from the University of Genova (Italy).

** Chercheur Associé au C.N.R.S. (C.P.T./Marseille).

[1] A wide review of the principles and applications of current algebra is found in Ref. [3].