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A Correction to My Paper Spectra of States, and Asymptotically Abelian C^*-Algebras

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Erling Størmer

Department of Mathematics, University of Oslo, Blindern, Oslo, Norway

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It was pointed out to me by Daniel Kastler that I was too careless in the use of the strong-* topology in the proof of Theorem 2.3 in the above paper [1]. As a result it is necessary to change the definition of the spectrum of a state on a C^*-algebra somewhat.

Definition 1. Let $\mathfrak{A}$ be a C^*-algebra and ϱ a state of $\mathfrak{A}$ with *GNS* representation $(\pi_\varrho, x_\varrho, \mathfrak{X}_\varrho)$. Then the *spectrum* of ϱ, denoted by $\mathrm{Spec}(\varrho)$ is the set of real numbers u such that given $\varepsilon > 0$ there is $A \in \pi_\varrho(\mathfrak{A})''$ for which $\omega_{x_\varrho}(A^*A) = 1$ such that

$$|u(\pi_\varrho(B)\,A x_\varrho, x_\varrho) - (A\pi_\varrho(B)\,x_\varrho, x_\varrho)| < \varepsilon\varrho(B^*B)^{1/2}$$

for all $B \in \mathfrak{A}$.

In the previous definition we asserted that we could choose $A \in \pi_\varrho(\mathfrak{A})$.

Let $\mathfrak{R}_\varrho$ denote the von Neumann algebra $\pi_\varrho(\mathfrak{A})''$ and E_ϱ the projection $[\mathfrak{R}'_\varrho x_\varrho]$, which is the support of ω_{x_ϱ} on $\mathfrak{R}_\varrho$. Let Δ_ϱ be the modular operator of x_ϱ relative to $E_\varrho \mathfrak{R}_\varrho E_\varrho$ acting on $E_\varrho \mathfrak{X}_\varrho$, and consider it as an operator on $\mathfrak{X}_\varrho$ by defining it to be 0 on $(I - E_\varrho)\,\mathfrak{X}_\varrho$.

Definition 2. With the above notation we call Δ_ϱ the *modular operator* of the state ϱ.

Remark 1. $\mathrm{Spec}(\varrho) = \mathrm{Spec}(\omega_{x_\varrho} | \mathfrak{R}_\varrho)$. Indeed, if $u \in \mathrm{Spec}(\varrho)$ and $A \in \mathfrak{R}_\varrho$ satisfies the conditions in Definition 1 then for all $B \in \pi_\varrho(\mathfrak{A})$

$$|u(A x_\varrho, B^* x_\varrho) - (B x_\varrho, A^* x_\varrho)| < \varepsilon \, \|B x_\varrho\| \, .$$

Since $\pi_\varrho(\mathfrak{A})$ is strong-* dense in $\mathfrak{R}_\varrho$ the same inequality holds for all $B \in \mathfrak{R}_\varrho$, and thus $u \in \mathrm{Spec}(\omega_{x_\varrho} | \mathfrak{R}_\varrho)$. The converse inclusion is trivial since $\pi_\varrho(\mathfrak{A}) \subset \mathfrak{R}_\varrho$.

Theorem. *Let* $\mathfrak{A}$ *be a* C^**-algebra and* ϱ *a state of* $\mathfrak{A}$ *with modular operator* Δ_ϱ. *Then* $\mathrm{Spec}(\varrho) = \mathrm{Spec}(\Delta_\varrho)$.