Commun. math. Phys. 38, 331—340 (1974)

Differentiability of Spatially Homogeneous Solutions of the Boltzmann Equation in the Non Maxwellian Case

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Received February 23, 1974

Abstract. The non linear Boltzmann equation is studied and differentiable solutions are shown to exist if the initial datum is suitably chosen

1. Introduction

In the present paper we study the initial value problem for the Boltzmann equation in the spatially homogeneous case and obtain an existence and uniqueness theorem which is a generalization of [4] when the extrinsic force is zero.

This space-independent problem has been much studied; for instance in [9] and [1] Povzner and Bodmer obtain a similar result by completely different methods. We note however that our methods are much more simple; moreover Povzner does not investigate the differentiability of the solution while Bodmer considers a somewhat less general equation.

2. Preliminaries

We begin with some basic definitions and results which will be used in the rest of this paper.

Let X be a real Banach space and X^* its dual; by $\| \ \|$ we denote the norm in X and by $\langle x, x^* \rangle$ the value of $x^* \in X^*$ at $x \in X$.

With each $x \in X$ is associated the set $\partial\|x\| = \{x^* \in X^*, \|x+y\| \geqq \|x\| + \langle y, x^* \rangle, \forall y \in X\}$; the application $X \to 2^{X^*}$, $x \to \partial\|x\|$, is called the subdifferential of the norm.

Let $f : D_f \subset X \to X$ and consider the initial value problem

$$\begin{cases} \dfrac{du}{dt} = f(u) & t \in [0, T] . \\ u(0) = u_0 \end{cases} \tag{1}$$

A continuous function $u : [0, T] \to X$ is called a solution of (1) if it is differentiable in $[0, T]$ and satisfies (1).