Commun. math. Phys. 38, 317—330 (1974)

The Connection between the Energy-Momentum Tensor and the Tensor Field in Presence of a Mass Gap

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Received November 29, 1973; in revised from May 10, 1974

Abstract. The conditions under which a tensor field can be regarded as an energy-momentum tensor are discussed. The problem connected with dilatational and conformal symmetries are exhibited.

1. Introduction

In most of the Lagrangean theories of relativistic fields it is assumed that the trace of the energy-momentum tensor is proportional to the square of the mass of the particle. Therefore for a massless field the trace should vanish. The validity of this assumption in the case of free fields can be explicitly verified.

In this note we are going to show – without having recourse to Lagrangean theory – that in a theory of strictly interacting, quantal fields with a mass gap a symmetrical, local, Poincaré covariant, locally conserved tensor field with a vanishing trace can not be used as an energy-momentum tensor since it gives rise to vanishing generators of the Poincare group.

This result can be used as an criterion to detect massless particles in the theory: the vanishing of the trace of the energy-momentum tensor entails the existence of massless particles.

We are going also to show that the dilatational current built with help of this tensor field yields a vanishing charge.

Contrary to the naive intuitive judgment the conformal current gives rise to a vector charge which does not always vanish and coincides with the energy-momentum vector. This result is verified by inspection on the model of scalar fields. It gives a prescription how to build an energy-momentum vector out of traceless tensor fields.

Finally we give the necessary and sufficient condition which have to be fulfilled by a tensor field in order that it can be used as an energy-momentum tensor.