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Billiards and Bernoulli Schemes

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Abstract. Some two dimensional billiards are Bernoulli flows.

0. Introduction

"Billiards" will mean a point particle moving on a table with smooth convex obstacles and bouncing elastically against them. Upon collision with the boundary the particle is either elastically reflected (reflecting billiards) or disappears to reappear at the opposite side (periodic billiards).

The qualitative theory of the above motion leads, in a natural way, to consider the flow S_t on the particle's phase space endowed with the Liouville measure.

Some simple questions can be answered if it is known that the flow S_t is ergodic.

Recently Sinai has given a proof that S_t is not only ergodic but, also, a K-flow. In this paper, making use of the results and techniques of Refs. [1–3], we prove that S_t is a Bernoulli flow.

We mention, however, that the knowledge that S_t is a K-flow or a B-flow is not sufficient to answer many questions of direct physical interest: consider, for instance, the average (with respect to the Liouville measure) of the cosine of the angle between the particle's velocity at time zero and its velocity at time t. How fast does it go to zero when t tends to infinity? [4].

The fact that it goes to zero is implied by the K-property (actually mixing would suffice): the B-property does not teach more about this problem and it seems that much work has still to be done to obtain other relevant information [4].

The reader will be assumed familiar with the definitions and the ideas of the paper in Ref. [3] which is necessary to understand without pain the thread behind the lemmas of Section 4.