Commun. math. Phys. 37, 327—329 (1974)

The Geometry of the (Modified) GHP-Formalism*

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Received May 11, 1974

Let (L, N) be a *pair of future oriented null direction fields* in a temporally and spatially oriented spacetime (M, g_{ab}) with a spinor structure [1–4]. Then the collection of null-tetrads $\zeta = (l, n, m, \bar{m})$ (as defined in the preceding paper) with $l \in L$, $n \in N$ is a *principal fibre bundle over M with structure group $\dot{C}$* (= multiplicative group of complex numbers), where, for $z \in \dot{C}$,

$$\zeta' = \zeta z \text{ means } (l', n', m') = \left(|z|^2 l,\, |z|^{-2} n,\, \frac{z}{\bar{z}} m \right). \tag{A.1}$$

Let B denote this bundle as well as its bundle space. B is a reduction of the bundle of oriented null tetrads over M ($\cong$ of oriented orthonormal frames).

If $\psi : M \to B$ is a cross section and (x^a) a local coordinate system of M, then (x^a, w) is a local coordinate system of B where, for $x \in M$, $\zeta_x \in B$, $w \in \dot{C} : \zeta_x = \psi_x w$. A complex valued 1-form $\bar{\omega}$ on B defines a *connection on B* if and only if it has the local representation

$$\bar{\omega} = \omega_a(x^b)\, dx^a + \frac{dw}{w}. \tag{A.2}$$

We then have $\psi^* \bar{\omega} = \omega_a \mathrm{d}x^a = \omega_\psi$, a 1-form on M depending on ψ and describing the connection relative to the tetrad field ψ. The curvature form is given by $d\bar{\omega}$ (on B) or by $\psi^* d\bar{\omega} = d\omega_\psi$ (on M).

A map η which associates with each cross section ψ of B a complex valued function η_ψ such that, for each map $z : M \to \dot{C}$,

$$\eta_{\psi z}(x) = z_x^p \bar{z}_x^q \eta_\psi(x)\,, \tag{A.3}$$

where (p, q) is a pair of integers, is said to be a *quantity of type* (p, q). If η is of type (p, q), its complex conjugate $\bar{\eta}$ is of type (q, p). The quantities of a definite type (p, q) form a C vector space, the quantities of all types together form a graded algebra $\mathfrak{A}$.

* This note should be considered as a supplement to the preceding paper by A. Held.