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Some Applications of Dilatation Invariance to Structural Questions in the Theory of Local Observables

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Abstract. In a dilatation-invariant theory it is shown that there is a unique locally normal dilatation-invariant state. Furthermore a gauge transformation of a local algebra cannot be implemented by a unitary operator from the local algebra. If the local field algebras are factors then so are the local observable algebras. The superselection structure of the theory can be determined locally.

1. Introduction

Various structural features of the algebra of local observables depend on the short-distance behaviour of the theory. Our aim is to study certain of these features by looking at what has become known as the Gell-Mann Low limit of the theory in hommage to the work of Gell-Mann and Low on the high-energy behaviour of quantum electrodynamics [1]. Roughly speaking the Gell-Mann Low limit of a theory is a dilatationally invariant theory with the same short-distance behaviour as the original theory.

In this paper we shall simplify matters by discussing dilatationally invariant theories; such theories are, so to speak, their own Gell-Mann Low limits. This enables the reader to focus his attention on the way the singular short-distance behaviour shapes the theory without being distracted by the technical assumptions on how the theory attains its Gell-Mann Low limits.

As an example of how structural features of the algebra of local observables can depend on the short-distance behaviour of the theory, consider the problem of whether one can measure the total electric charge contained within a sphere of radius R by means of an observation inside the sphere. Formally the operator

$$Q_R = \int\limits_{|\boldsymbol{x}| \leqq R} j_0(0, \boldsymbol{x})\, d^3\boldsymbol{x}\,, \tag{1.1}$$