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Parametric Interactions and Scattering

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Abstract. We show how a variety of parametric Hamiltonians arise by a limiting procedure applied to a time-independent Hamiltonian. We then study one such Hamiltonian, that for a parametric frequency converter, in detail and find its associated Raman scattering matrix.

1. Introduction

We study the time evolution of a system with the Hamiltonian

$$H = H_0 + \omega J_3 + \frac{\lambda}{N}(J_- X + J_+ X^*) \tag{1.1}$$

defined on $\mathbb{C}^N \otimes \mathscr{F}$ where

(i) the operators J_3, $J_\pm$ on $\mathbb{C}^N$ satisfy the commutation relations

$$[J_3, J_\pm] = \pm J_\pm ; \qquad [J_+, J_-] = 2J_3 . \tag{1.2}$$

(ii) H_0 is a self-adjoint operator on the Hilbert space $\mathscr{F}$;

(iii) X is an operator on $\mathscr{F}$ of a suitably regular type.

We take the initial state on $\mathbb{C}^N$ to be a pure superradiant state, that is

$$\varrho_A = |\xi_N\rangle\langle\xi_N| \tag{1.3}$$

where

$$J_3 \xi_N = \gamma_N N \xi_n ; \qquad -\tfrac{1}{2} < \lim_{N\to\infty} \gamma_N \equiv \gamma < \tfrac{1}{2} . \tag{1.4}$$

If ϱ is the initial mixed state on $\mathscr{F}$ then the state at time t is defined by

$$T_t^{(N)}(\varrho) = \mathrm{tr}_{\mathbb{C}^N}[e^{-iHt}(\varrho_A \otimes \varrho) e^{iHt}] \tag{1.5}$$

this being a density matrix on $\mathscr{F}$. We are interested in finding the limit of this as $N \to \infty$. The limit if it exists is written as

$$T_t(\varrho) = \lim_{N\to\infty} T_t^{(N)}(\varrho) \tag{1.6}$$

and is for each t a positive trace-preserving linear map on the space of all density matrices on $\mathscr{F}$.