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On the Local Central Limit Theorem for Gibbs Processes

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Abstract. We derive a sufficient condition for the validity of the local central limit theorem for Gibbs processes and their isomorphism with a Bernoulli shift.

1. Introduction

It has been recently realized to the class of δ-dimensional discrete time stationary Markov processes and the class of translation invariant finite range Gibbs processes are in one-to-one correspondence and define the same class of random fields [1–3] (see below for a precise statement).

To fix the notations we briefly recall the definitions. For simplicity the space of states will be restricted to be $I = [0, 1]$: the generalization to I finite is straightforward.

Definition 1. A stationary Markov process on a δ-dimensional lattice Z^δ is

i) a translation invariant Borel probability measure μ on I endowed with the product topology (I being considered with the discrete topology);

ii) μ has the property that if $\Lambda \subset Z^\delta$ is a finite region then the probability distribution of the events inside Λ is independent on the events outside

$$\Lambda_{\mathscr{d}} \equiv \{\xi/\xi \in Z^\delta/\Lambda, d(\xi, \Lambda) \leqq \mathscr{d},\ d(\xi, \Lambda) = \text{distance of } \xi \text{ from } \Lambda\}$$

where $0 \leqq \mathscr{d} < \infty$ and $\mathscr{d}$ depends on μ but not on Λ.

Before defining a Gibbs process observe that I^{Z^δ} can be regarded as the set of subsets $X \subset Z^\delta$. If $X \in I^{Z^\delta}$ is regarded as a subset of Z^δ we shall call its points the "occupied points" and we shall refer to X as to a "configuration" [1].

[1] It would be more appropriate to call such a process a d-Markov process I or a finite memory process since, if $\delta = 1$, it does not reduce to the equal definition of Markov process (unless $\mathscr{d} = 1$). However it is known that one-dimensional finite memory processes are Markov processes on a different space of states (see, for istance, [1]).