

A Proposition-State Structure

I. The Superposition Principle

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Abstract. A generalization of the superposition principle of quantum mechanics is proposed introducing the concept of maximal state of a logic.

1. Introduction and Basic Axioms

It is generally admitted that the superposition principle (in the sense of Dirac's book) is the point where the departure of quantum theory from the classical physics is more evident. In order to have a precise mathematical formulation of this principle, it is particularly convenient the language of lattice theory where the difference between classical and Quantum theories can be made very transparent. There exist at present, many expositions of the lattice approach [1, 2]. In view of the formulation of the superposition Principle, the Varadarajan's framework is of particular interest [2]. There it is given a notion of superposition of states which includes both the concept of classical mixture of states as well as the concept of quantum superposition of states.

In this way it is open the possibility of the existence of (non trivial) pure superpositions of pure states. This situation comes out, of course, specializing the lattice structure, to get, via Piron's theorem [1 a], the standard logic. In view of the special nature of the assumptions leading to a standard logic, it is of some interest the question if the above mentioned situation occurs in a more general context. In this paper we attempt to give a positive answer to that question.

We associate to the physical system a pair (L, S) : the logic L represents the set of all the classes (propositions) of equivalent yes-no experiments and the set S represents the set of all the preparing procedures pertaining to the physical system. The mathematical assumptions on (L, S) are the following. The set L has the structure of complete, orthocomplemented

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