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Asymptotic Behaviour of Eigenfunctions for Multiparticle Schrödinger Operators

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Abstract. O'Connor's approach to spatial exponential decay of eigenfunctions for multiparticle Schrödinger Hamiltonians is developed from the point of view of analytic perturbations with respect to transformation groups.

This framework allows an improvement of his results in some directions; in particular if interactions are dilation analytic, exponential fall-off is shown to hold for any bound-state wave-function corresponding to an eigenvalue distinct from thresholds; it is shown that the exponential decay rate depends on the distance from the bound-state energy to the nearest threshold. Applications include non existence of positive energy bound-states.

Exponential fall off for eigenstates of many-particle Schrödinger Hamiltonians has been investigated in detail in a recent work of O'Connor [1]. He assumes that particles interact through multiplicative two-body potentials in $R + L^\infty_\varepsilon$ (Rollnik class plus arbitrarily small L^∞ tail); then if E_0 is the lowest threshold of the system, bound-state wave-functions associated to an eigenvalue E with $E < E_0$, are in the domain of $e^{\theta \sqrt{2M(E_0 - E)} R}$, $0 < \theta < 1$; here M is the total mass of the system and R is the radius of gyration operator about the centre of mass of the system. Other results on the asymptotic behavior of eigenfunctions for Schrödinger Hamiltonians include [1] Bazley and Fox, for one electron molecular ions; Faris, single particle Hamiltonians with exponentially decaying potentials; Alrichs, multiparticle atomic systems with Coulomb interactions.

These results supplement the smothness properties for multiparticle eigenfunctions described by Kato [2] and have important theoretical implications for bound-state or scattering problems. However they pertain to eigenfunctions with isolated eigenvalues of Schrödinger Hamiltonians with local interactions. These two constraints are not satisfactory from a mathematical point of view and not physically justified (e.g. existence of permutation or rotation symetries can lead to bound-states in the continuum).

Our major aim is to show how one can avoid such restrictions and obtain a general treatment of exponential fall-off with the help of analytic