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The Infinite Atom Dicke Maser Model II

E. B. Davies

Mathematical Institute, Oxford, U.K.

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Abstract. We study the time evolution of a quantum field under a Hamiltonian constructed in an earlier paper by taking the limit as $n \to \infty$ of a Dicke maser model Hamiltonian for n radiating atoms. We show that the radiation field converges to a dynamic equilibrium state independent of its initial state and that the strength of the field is inversely proportional to the square of the distance from the source. A number of variations of the Hamiltonian are also considered.

1. Definition of the Hamiltonian

In an earlier paper [2] we studied the limit as $n \to \infty$ of a sequence of Dicke maser model Hamiltonians H_n on the spaces

$$\{\otimes^n \mathbb{C}^2\} \otimes \mathscr{F} \tag{1.1}$$

where $\mathscr{F}$ is a Boson Fock space. The Hamiltonian H_n describes a simple interaction between n 2-level atoms and a quantum field with an infinite number of degrees of freedom. The limiting Hamiltonian H was realised on

$$l^2(\mathbb{Z}) \otimes \mathscr{F} \simeq L^2\{(-\pi, \pi), \mathscr{F}\}. \tag{1.2}$$

In this paper we study the time evolution for the limiting Hamiltonian. This is done in substantially greater generality than is required for the development of [2]. The reason for this is that we wish to be able to treat a number of variations of the maser model – for example the case of multi-level atoms with a number of different emission modes.

We start by describing the quantum field in terms of a representation of the canonical commutation relations. We take a complex test function space D dense in the single particle Hilbert space D^-; D is supposed to be a complete locally convex topological linear space under a topology stronger than the Hilbert space topology. The single particle Hamiltonian S is supposed to be essentially self-adjoint on D and the unitary group e^{iSt} is supposed to leave D invariant and to be jointly continuous from $\mathbb{R} \times D$ to D. The quantum field is defined on a Hilbert space $\mathscr{K}$ by a representation of the C.C.R.'s on $\mathscr{K}$. For each $f \in D$ there is a unitary