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Non Quasi-free Classes of Product States of the C.C.R.-Algebra

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Abstract. Two exemples of pure states of Van Hove's Universal Receptacle in the boson case are presented with are not unitarily equivalent to any quasi-free state. In particular, it is shown that a discrete state is unitarily equivalent to some quasi-free state if and only if it is equivalent to the Fock state related to the chosen decomposition of the test function space.

I. Introduction

This paper is a continuation of a previous one [1] in which we showed that non discrete pure states of the Van Hove's Universal Receptacle in the fermion case are not unitarily equivalent to any quasi-free state. The situation in the boson case is a little more complicated. Indeed, the quasi-free states of the C.C.R.-algebra are of discrete and non discrete type [2]. If we restrict ourselves to a fixed basis of the test function space H_0 the discrete states are equivalent to a class of states which we called "physically pure" ones. Those "physically pure" states are different from the quasi-free states except for the Fock state, moreover there exist[1] non discrete states which are disjoint from every quasi-free state of the decomposition of H_0 we consider. But the question remained open as if we can state the same assertions considering all quasi-free states issued from any possible decomposition of H_0.

I.1. Notations

Let $(H_k)_{k\in\mathbb{N}}$ a countable family of two-dimensional real vector spaces, and $H=\bigoplus_{k\in\mathbb{N}} H_k$ the weak sum of the H_k's. ($H=\{\varphi\in H_0 \mid P_k\varphi=0$ for a finite number of k's$\}$, $H_0=\bigoplus_{k\in\mathbb{N}} H_k$ denoting the Hilbert sum).

Equipped with σ, a regular, antisymmetric, real bilinear form (H_0,σ) is a separable symplectic space.

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[1] The Klauder-McKenna-Woods criterion [3] provides examples of this, as $\Omega_k=1/\sqrt{2}\,\xi_k^1+1/\sqrt{2}\,\xi_k^2$. See notation further.