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# Lorentz Covariance of the $P(\varphi)_2$ Quantum Field Theory without Higher Order Estimates

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**Abstract.** We give a simple proof of Lorentz covariance for the $P(\varphi)_2$ model without using the higher order estimates: For each Poincaré transformation $\{a, \Lambda\}$ and each bounded region $B$ of Minkowski space there exists a unitary operator $U$ which correctly transforms the Heisenberg picture field operator: $U\varphi(f)\,U^* = \varphi(f_{\{a,\Lambda\}})$, $f \in C_o^\infty(B)$.

## I. Introduction

The Lorentz covariance of boson field theories in two dimensional space-time was first studied by Cannon and Jaffe [1] for the $(\varphi^4)_2$ model in the sense of Haag-Kastler axioms [4]. Their results were extended to the $P(\varphi)_2$ by Rosen [9]. In each case, higher order estimates were used to study the corresponding models. It is well known that most of the results for the $P(\varphi)_2$ model can be obtained by using the hypercontractive property of the semi-group $e^{-tH_0}$ [2, 3, 5, 11]. Recent results by Klein have shown the self-adjointness of the locally correct generator of Lorentz transformation for the $P(\varphi)_2$ interaction by introducing the $L_2(Q, d\mu)$ representation of Fock space $\mathscr{F}$ [7].

The main purpose of this paper is to simplify the proof of Lorentz covariance for the $P(\varphi)_2$ interaction by using the hypercontractive properties of the semigroups generated by the locally correct Hamiltonian and Lorentzian. We shall follow the method developed by Cannon and Jaffe [1]. However, we are able to prove the main theorems of references [1] and [9] using only hypercontractive semi-groups; we don't use the higher order estimates.

The locally correct Hamiltonian we shall consider has the form

$$H(g) = H_0 + H_I(g) \tag{1.1}$$

with

$$H_I(g) = \int : P(\varphi(x)) : g(x)\, dx\,, \tag{1.2}$$