Commun. math. Phys. 34, 149–166 (1973)

On Conformal Invariance of Interacting Fields

W. Rühl

Fachbereich Physik, Universität Trier-Kaiserslautern, Kaiserslautern, Federal Republic of Germany

Received July 13, 1973

Abstract. We study the action of the conformal algebra on interacting fields. On a certain set of states the algebra is integrated to projective representations of $SU(2,2)$. These representations are shown to be equivalent to the representations of the interpolated discrete series of $SU(2,2)$. Using this result we give a formula for the two-point Wightman function for arbitrary spin and dimension of the field. Finally we discuss the limit when the dimension tends to the canonical value.

1. Notations and Summary of Results

We consider spinorial fields

$$\Phi_{A\dot{B}}(x) \quad \text{or} \quad \Phi_A{}^{\dot{B}}(x) = (-1)^{j_2 - B}\,\Phi_{A,-\dot{B}}(x)$$

and assume the existence of a unitary representation of the inhomogeneous proper orthochronous Lorentz group (Poincaré group) satisfying

$$\begin{aligned} & U(y,\Lambda)\,\Phi_A{}^{\dot{B}}(x)\,U(y,\Lambda)^{-1} \\ & = \sum_{A'B'} D^{j_1}_{AA'}(a^{-1})\,D^{j_2}_{\dot{B}\dot{B}'}(a^\dagger)\,\Phi_{A'}{}^{\dot{B}'}(x') \end{aligned} \tag{1.1}$$

with

$$x' = \Lambda x + y, \qquad \Lambda = \Lambda(a)\,. \tag{1.2}$$

By $\Lambda = \Lambda(a)$ we denote the well known two-to-one homomorphism between $SL(2,C)$ and the proper orthochronous Lorentz group

$$\begin{aligned} & X = x^0\sigma_0 - \sum_i x^i\sigma_i, \qquad X' = aXa^\dagger \\ & \hat{X} = x^0\sigma_0 + \sum_i x^i\sigma_i, \qquad \hat{X}' = a^{-1\dagger}\hat{X}a^{-1} \\ & x'^\mu = \Lambda^\mu{}_\nu x^\nu\,. \end{aligned} \tag{1.3}$$

We assume moreover that operators K_μ and D exist that together with

$$P_\mu = -i\frac{\partial}{\partial y^\mu}\,U(y,\mathbb{1})|_{y=0}\,, \tag{1.4}$$

$$M_{\mu\nu} = -i\frac{\partial}{\partial\omega^{\mu\nu}}\,U(0,\Lambda)|_{\omega^{\mu\nu}=0} \tag{1.5}$$