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Hamiltonian Formalism for Non-invariant Dynamics*

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Abstract. In a completely Hamiltonian dynamical system, there will be a generating function H_Y for each infinitesimal space-time transformation Y. In the non-autonomous case, the H_Y depend on the observer. This dependence is here described by a system of commutation relations. It is also shown that these relations can be made to mirror exactly the commutation relations of the Y's in the Lorentz-invariant case.

1. Introduction

A system involving an external field can be enlarged to a completely autonomous system whenever a transformation-law for the field is specified. By a method presented below we can enlarge any non-autonomous system to an "augmented" system which is completely autonomous. When the original system preserves some Hamiltonian structure, an analysis of the augmented system leads to the results stated about the H_Y. The systems considered are of the classical type in that there are only finitely many degrees of freedom but are more general in that the entire space-time group, and not merely temporal changes as in classical dynamics, are allowed as changes in observes.

For any given infinitesimal change of observer, the generating function H_Y is almost unique: unique up to an additive term independent of the dynamical coordinates. If such terms are improperly adjusted, then the relation $\{H_Y, H_Z\} = H_{[Y,Z]}$, expected in the invariant case, may not hold. In the Lorentz-invariant case we show that these relations can be achieved.

2. Space-time, Coordinators, and Dynamics

For any discussion of dynamical systems, one must have a space-time manifold M, and M must have a space-time structure. The latter includes (usually tacitly) a differentiable-manifold structure (usually 4-dimen-

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