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Infrared Singularities and Small-Distance-Behaviour Analysis

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Abstract. The infrared-singularity structure of the vertex functions of massless-particle ϕ^4 theory is studied. This allows to construct the asymptotic forms of the vertex functions of massive-particle ϕ^4 theory in a simpler and more explicit fashion than in a previous paper. With the help of the parquet approximation introduced by Diatlov, Sudakov, and Martirosian we show that the infrared-singularity structure in a theory with besides the massless particles, massive ones is the same as in the theory with massless particles only. All these results in ϕ^4 theory have analoga in other renormalizable theories.

Introduction

In a series of papers [1] a systematic approach to the large-momenta problem for vertex functions[1] (VFs) in renormalizable field theories has been undertaken. A main step hereby was to define the asymptotic forms (AFs) of those functions. At generic momenta, the AFs are the VFs of a corresponding zero-mass theory, and their behaviour under overall scaling of the momenta is described by the renormalization group equations for such a theory, given simplest in the form of homogeneous partial differential equations (PDEs) obeyed by these functions.

The momenta sets at which the zero-mass theory VFs are infrared (UR) singular are called exceptional. The AFs of the massive-theory VFs at Euclidean such momenta are expressible as certain UR finite parts extracted from the zero-mass theory VFs at those momenta, and transform in a fashion, different from case to case, given simplest in terms of certain (in general) inhomogeneous PDEs, the inhomogeneous terms involving other exceptional AFs.

In the formulae derived in Appendix B of SD 2, for the described connection between exceptional finite-mass theory AFs and zero-mass theory VFs, coefficient functions appeared for which definitions only as limites were given. In this paper, we derive equivalent but simpler formulae, with explicit expressions for the coefficient functions. These formulae are developed here for ϕ^4 theory, but have analoga in all renormalizable theories.

[1] The amputated one-particle-irreducible parts of connected Green's functions.