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On Clustering States

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Abstract. Clustering properties of states over the algebra of local observables are discussed under the weak form of "asymptotic abelianness".

1. Introduction

In the framework of the algebraic approach to the quantum field theory and statistical mechanics, we have various notions about locality called "asymptotic abelianness" ([4]–[11]). All of them are the constraints imposed upon the algebra of local observables, under which clustering properties of states over the algebra are discussed.

These "algebraic" conditions seem to be too restrictive for characterizing "states". Besides, though the conditions are reasonable when the automorphism group consists of space translations, we may not expect algebraic "asymptotic abelianness" in the case of time translations. So in this note, we require "asymptotic abelianness" as restrictions over states. Our conditions are much weaker than algebraic ones and we can still discuss clustering properties of states under them.

2. Strongly Clustering State

Let $\mathfrak{A}$ be a C^*-algebra, τ a mapping from an infinite set G into automorphisms of $\mathfrak{A}$ and S the set of states on $\mathfrak{A}$. For any ϕ in S there correspond a Hilbert space H_ϕ, a representation π_ϕ of $\mathfrak{A}$ on H_ϕ and a unit cyclic vector Ω_ϕ such that

$$\phi(a) = (\Omega_\phi, \pi_\phi(a)\, \Omega_\phi), \qquad a \in \mathfrak{A}.$$

Let E_Ω be the projection on vector Ω_ϕ.

Definition 1. A state $\phi \in S$ is called

i) *strongly G-central iff for any $a, b \in \mathfrak{A}$*

$$\lim_{g \to \infty} \phi([\tau_g(a), b]) = 0$$

and

ii) *strongly clustering iff for any $a, b \in \mathfrak{A}$*

$$\lim_{g \to \infty} |\phi(\tau_g(a)\, b) - \phi(\tau_g(a))\, \phi(b)| = 0.$$