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Ising Model and Bernoulli Schemes in One Dimension

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Abstract. We prove that the one-dimensional random fields with finite first moment are isomorphic to Bernoulli schemes.

§ 1. Introduction and Notations

We consider stationary processes defined on a space I of states with only two elements: $I = \{0, 1\}$. A process will thus be a regular probability measure μ defined on the compact space $K_Z = \prod_{i \in Z} I$, where Z is the set of the integers and K_Z is considered with the topology product of the discrete topologies on the factors I.

The elements of K_Z will be identified with the subset $X \subset Z$.

If $\Lambda \subset Z$ is a finite set and $K_{Z/\Lambda}$ is defined in analogy with K_Z, (Z/Λ is the complement of Λ), the process defines a natural measure μ_Λ on $K_{Z/\Lambda}$ and a natural probability distribution f_Λ on the set of subsets of Λ:

$$\mu_\Lambda(E) = \sum_{X \subset \Lambda} \mu(X \cup E) \qquad \forall\, E \subset K_{Z/\Lambda}, \tag{1.1}$$

$$f_\Lambda(X) = \mu(\{Y/Y \in K_Z, Y \cap \Lambda = X\}) \qquad \forall\, X \subset \Lambda. \tag{1.2}$$

Notice that $\{Y/Y \in K_Z, Y \cap \Lambda = X\}$ can be thought as an atom $A(\Lambda, X)$ of the partition $\bigvee_{i \in \Lambda} T^i P$ where T is the shift operator (rightwards) on K and $P = (P_0, P_1)$ is the two set (generating) partition of K_Z consisting in the sets:

$$\begin{aligned} A(\{0\}, \emptyset) &= \{Y/Y \in K_Z, Y \cap \{0\} = \emptyset\} \\ A(\{0\}, \{0\}) &= \{Y/Y \in K_Z, Y \cap \{0\} = \{0\}\}. \end{aligned} \tag{1.3}$$

Stationarity of the process means that $f_\Lambda(X) = f_{\Lambda+s}(X+s)$ where $X + s = (x_1 + s, x_2 + s, \ldots)$ if $X = (x_1, x_2, \ldots)$ and $s \in Z$.

If μ is a process we can define the conditional probabilities $f_\Lambda(X/Y)$, for $X \subset \Lambda$, Λ finite, $Y \subset Z/\Lambda$, as the conditional probability for finding

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