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Existence and Uniqueness of Equilibrium States for Some Spin and Continuum Systems

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Abstract. The one to one correspondence between the existence of a unique equilibrium state and the differentiability of the free energy density with respect to the external field previously shown for Ising ferromagnets is extendend to higher valued spin systems as well as to continuum systems satisfying the Fortuin, Kasteleyn and Ginibre inequalities. In particular this is shown to hold for a mixture of $A-B$ particles in which there is no interaction between like particles and a repulsion between unlike particles. Where the derivative of the free energy is discontinuous there are at least two equilibrium states.

1. Introduction

In a previous paper [1] we considered lattice spin systems with Hamiltonians

$$H = -\sum_{i<j} J(i,j)\, S_i S_j - \sum_{i<j} \gamma(i,j)\, S_i^2 S_j^2 - \sum_i h(i)\, S_i - \sum_i \mu(i)\, S_i^2 \quad (1.1)$$

where the summation is over all sites of the lattice Z^ν with spacing δ, contained in a region $\Lambda \subset R^\nu$, and S_i, the spin variable at the ith site can take on the integer values $p, p-2, \ldots, -p+2, -p$. For such a system it was shown that the FKG inequalities [2] hold whenever

$$J(i,j) \geqq (2p-2)^2\, |\gamma(i,j)|\,, \quad \text{for all} \quad i,j \in \Lambda \quad (1.2)$$

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