

# Analyticity Properties of the Ising Model in the Antiferromagnetic Phase

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**Abstract.** The partition function of the Ising antiferromagnet is proved to have no zeroes in an annulus around the origin in the complex  $z$ -plane. The intersection of this annulus with the positive real axis belongs to the antiferromagnetic region. The free energy and the correlation functions are analytic in the annulus.

## Introduction

In the present article we study the cubic Ising model with repulsive nearest neighbour interaction in the antiferromagnetic region. For simplicity we take the two dimensional case, but the results remain true in higher dimensions.

All points  $x$  of the two dimensional lattice  $\mathbb{Z}^2$  can have spin  $\sigma_x = \pm 1$ . To a finite volume  $A$ , a configuration  $\sigma = \{\sigma_x, x \in A\}$  in  $A$  and a boundary condition  $\tau = \{\tau_x, x \notin A\}$  we assign the energy

$$H_A(\sigma|\tau) = J \sum_{\substack{\langle x,y \rangle \\ x,y \in A}} \sigma_x \sigma_y - h \sum_{x \in A} \sigma_x + J \sum_{\substack{\langle x,y \rangle \\ x \in A, y \notin A}} \sigma_x \tau_y. \quad (1)$$

In this expression,  $\langle x, y \rangle$  denotes the summation over pairs of nearest neighbours; repulsive interaction means that  $J > 0$ .

For low temperatures and small absolute values of the magnetic field  $h$ , the system is known to have at least two equilibrium states [3]. They can be obtained as limits of finite volume Gibbs distributions with the boundary conditions

$$\tau = \pm \varepsilon, \quad (2)$$

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