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The Use of Unitarity Bounds for a Stable Extrapolation of Low Energy Data

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Abstract. The properties of the scattering amplitude allow to define a function $f(z)$ satisfying the following conditions:

1) $f(z)$ is holomorphic in a simply connected domain $\mathscr{D}$, which can be mapped conformally onto the unit disk;

2) $|\mathrm{Im}\, f(z)|$ is bounded by some constant M in $\mathscr{D}$;

3) $|\mathrm{Re}\, f(z)|$ is known not to exceed some constant m on a certain part Γ_1 of the boundary Γ of $\mathscr{D}$; $f(z)$ is continuously extensible onto Γ.

Using these properties, constraints are derived on the real part of $f(z)$ valid at any point $z \in \mathscr{D} \cup \Gamma$.

The result is used for performing a stable extrapolation of low energy pion-pion scattering data to any finite energy. We derive a bound on energy averaged values of the real part of the scattering amplitude. The bound depends on m, M, on the energy variable s and on the energy average interval $s_2 - s_1$.

Generalizations of the method are discussed.

1. Introduction

Upper and lower bounds on total and elastic cross-sections, scattering amplitudes, phases, etc., represent an important part of our exact knowledge in strong interaction physics. Since the pioneering work of Froissart [1], great progress has been made during the past decade, mainly due to the work of Martin and his collaborators. Important results have been obtained, especially in the following respects:

1) assumptions required for obtaining some bounds have been weakened;

2) some of the more recent bounds approach experimental curves rather closely;

3) a number of upper bounds on the *averages* of total cross-sections over finite energy intervals have been derived.

Details can be found in reviews [2–9] containing further references.

The average bounds are of particular interest because arbitrary unknown constants usually occurring in the bounds can be eliminated

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