Commun. math. Phys. 30, 83—98 (1973)

Time Evolution of a Quantum Lattice System

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Received October 5, 1972

Abstract. It is shown that for a quantum lattice system associated with a Hamiltonian with a kinetic part and a potential sufficiently decreasing in the particle number, the time evolution can be described, under certain assumptions, by automorphisms of a suitable algebra.

1. Introduction

It has been shown by Robinson [1] that the time evolution induced by the following quantum lattice system Hamiltonian:

$$H_\Lambda = \sum_{X \subset \Lambda} \Phi(X)$$

with

$$\sum_{X \ni 0} \|\Phi(X)\| \exp(|X| - 1) < +\infty \tag{1.1}$$

where $|X|$ is the number of sites in the region X, can be described by an automorphism of the quasi-local algebra associated with the lattice system.

Attempts to generalize this result to a wider class of potentials have been made afterwards. Ruskai [2] has shown that for a certain class of potentials, the time evolution is at least an automorphism of an algebra larger than the observables' algebra. The case of continuous systems is not yet clear. Ruelle [3] has proven the existence of the infinite volume Green functions for low density quantum gas.

Our purpose is to consider a lattice system described by a Hamiltonian not verifying (1.1) and to show that the time evolution is an automorphism of the weak closure of the observables' algebra in the G.N.S. representation generated by an equilibrium state at low density.

In Section 2 we combine the technique of [4] and [5] in order to show the existence of the reduced density matrices (R.D.M.) at infinite volume using Ginibre's method based on the Kirkwood-Salzburg integral equations.