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On the Purification Map

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Abstract. The investigation of purifications of factor states has been carried on. It is shown, that any factor state ω of a C^*-algebra admits at most one purification $\tilde{\omega}$, so one can introduce the purification map $\phi : \phi(\omega) = \tilde{\omega}$. It turns out, that the Powers and Størmer inequality is valid in this general situation.

0. Introduction

Let $\mathfrak{A}$ be a C^*-algebra and $\mathfrak{A}^\circ$ be an opposite algebra. It means, that $\mathfrak{A}^\circ$ is a C^*-algebra and that an antilinear, multiplicative, $*$-invariant isometry of $\mathfrak{A}$ onto $\mathfrak{A}^\circ$ is given. The image of an element $a \in \mathfrak{A}$ will be denoted by $\bar{a} \in \mathfrak{A}^\circ$. As in [7] we introduce

$$\tilde{\mathfrak{A}} = \mathfrak{A}^\circ \otimes \mathfrak{A}$$

where the tensor product is taken in the sense of the C^*-algebra theory (it includes a suitable completion such that $\tilde{\mathfrak{A}}$ becomes a C^*-algebra). We shall assume, that $\mathfrak{A}$ contains the unity 1 and shall identify any element $a \in \mathfrak{A}$ with $\bar{1} \otimes a \in \tilde{\mathfrak{A}}$. This way $\mathfrak{A}$ becomes a subalgebra of $\tilde{\mathfrak{A}} : \mathfrak{A} \subset \tilde{\mathfrak{A}}$.

In what follows, we shall consider only such states of C^*-algebras, which give rise (by G.N.S.-construction) to representations in separable Hilbert spaces.

Let us recall (see [7]), that a state $\tilde{\omega}$ of $\tilde{\mathfrak{A}}$ is said to be j-positive iff

$$\tilde{\omega}(\bar{a} \otimes a) \geqq 0\,, \qquad a \in \mathfrak{A} \tag{0.1}$$

Any such state is j-invariant i.e.:

$$\tilde{\omega}(j(\tilde{a})) = \overline{\tilde{\omega}(\tilde{a})} \tag{0.2}$$

for any $\tilde{a} \in \tilde{\mathfrak{A}}$. In the above equation j denotes the antilinear, multiplicative, $*$-invariant, involutive (i.e. $j^2 = \mathrm{id}$) mapping

$$j : \tilde{\mathfrak{A}} \to \tilde{\mathfrak{A}}$$

introduced by the formula

$$j(\bar{a} \otimes b) = \bar{b} \otimes a\,.$$