Commun. math. Phys. 30, 1—10 (1973)

Necessary and Sufficient Conditions for Current Conservation in the Framework of Relativistic Quantum Field Theory

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Received May 8, 1972; in revised form September 25, 1972

Abstract. Necessary and sufficient conditions for the conservation of vector currents covariant under translations and of the Conformal currents, are formulated and proven. Only the weak spectrum condition is assumed, thus the theorems serve as generalizations of a well-known theorem due to Coleman. This is done under the axioms of Relativistic Quantum Field Theory due to Wightman.

I. Introduction

In a previous publication [1], necessary and sufficient conditions for conservation of internal and conformal currents were formulated and proven. This treatment lacks rigour (e.g., distributions were integrated as ordinary functions). In the present treatment appropriate formulations and proofs are given in the framework of Relativistic Quantum Field Theory [2].

The theorems proven in this note serve as a rigorous formulation of the connection between current conservation and the fact that the corresponding charge annihilates (in some well defined manner) the vacuum state. This connection is known as "Coleman's Theorem" [3]. Our statements differ from previous versions of Coleman's Theorem as they are in a form of necessary and sufficient conditions and we do not demand a mass gap.

Thus, our theorems apply to cases where the presence of zero mass particles is an intrinsic feature of the symmetry group under consideration, as is the case with unbroken Chiral symmetry in the realization where the baryons are massless and unbroken Conformal symmetry where all the masses vanish. Furthermore, the theorems apply also to cases of spontanously broken internal and Conformal symmetries.

II. The Framework and Praeliminaria

The framework is Relativistic Quantum Field Theory. The postulates of the theory are given in Chap. 3 of Ref. 2. We stress that only the so