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The Ising Problem

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Abstract. A method to solve Ising problems is developed giving all correlation functions. As an example the one-dimensional nearest and next-nearest neighbour models have been calculated explicitly.

Introduction

One of the basic problems in statistical mechanics is to understand the phenomenon of a phase transition, i.e. why short range interactions can act like long range interactions or to put it otherwise: why is the cluster property destroyed? A quite logical approach to this problem is to solve models, which exhibit a phase transition. The conventional way is to calculate the partition function for a finite system, from which all thermodynamical properties are derived. The phase transition only occurs after taking the thermodynamic limit, because all thermodynamic functions are analytic in the temperature for a finite system [1], whereas the definition of a phase transition is that they should contain singular points. If one wants a better understanding of the nature of the phase transition it is necessary to know more about the equilibrium state of the system (i.e. all correlation functions) than only its thermodynamics. This knowledge can not be obtained from the partition function.

Up to now there is, except for the free gas, no complete description of any model in statistical mechanics. In the following we will present a method to calculate all equilibrium correlation functions of the simplest models in statistical mechanics, namely the one-dimensional Ising models with finite range interactions. The results for the nearest-neighbour and the next nearest-neighbour model are given. This technique is based on the topological and algebraic structure of the system. The system is taken to be infinite right from the beginning and its equilibrium state is obtained from the K.M.S. boundary condition [2]. We expect this method to be independent of the dimension of the lattice; it gives a finite closed set of algebraic equations for all finite-dimensional Ising systems with finite range interactions with or without magnetic field. For the