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The Field of Charged, Spinning, Magnetic Particles

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Abstract. Stationary solutions of the Einstein-Maxwell equations have recently been given corresponding to charged, spinning magnetic matter. In this paper a solution for two particles of such matter is given. In general it contains a singularity between the particles.

§ 1. Introduction

Perjés [1] and Israel and Wilson [2] have recently given independently a new class of stationary solutions of the Einstein-Maxwell equations. These solutions describe the exterior field of massive, charged, magnetised, spinning particles. The electric charges and masses of the particles are such that the electric and gravitational forces between any two of them balance, and in this respect the solutions resemble those of Papapetrou [3] and Majumdar [4]. However, the particles also possess magnetism and spin, and in relativistic units the magnetic moment is equal to the angular momentum. In fact, every particle satisfies, in relativistic units ($c=1$, $G=1$)

$$m=|e|, \quad \boldsymbol{h}=\pm\boldsymbol{\mu}, \tag{1.1}$$

m, e, $\boldsymbol{\mu}$ and $\boldsymbol{h}$ being the mass, charge, three dimensional magnetic moment and angular momentum.

In units of customary dimensions (1.1) become

$$m\sqrt{G}=|e| \quad \text{(a)}, \qquad c^{-1}\sqrt{G}\boldsymbol{h}=\pm\boldsymbol{\mu} \quad \text{(b)} \tag{1.2}$$

and objects with these parameters are physically quite plausible. For instance (a) is satisfied by a sphere of neutral hydrogen in which one atom in about 10^{18} has lost its electron; $|\boldsymbol{h}|/|\boldsymbol{\mu}|$ given by (b) is only about three orders of magnitude greater than the corresponding ratio for the Earth. Though objects satisfying both (a) and (b) together, even approximately, must be somewhat rare, it is clear that the matter described by the solutions is physically possible. It is therefore worth investigating them to see what insight they give into general relativity.

The solution for the simplest particle satisfying (1.1) was given by Perjés [1], and we propose to call it a *Perjeon.* If $\boldsymbol{h}=\boldsymbol{\mu}=0$ the particle