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Ultralocal Quantum Field Theory in Terms of Currents

I. The Free Theory

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Abstract. The paper considers the possibility of constructing ultralocal theories, whose Hamiltonians contain no gradient terms and are therefore diagonal in position space, entirely in terms of currents with an equal time current algebra replacing the canonical commutation relations. It is shown that the free current theory can be defined in terms of a certain representation of the current algebra related to the group, $SL(2, R)$. This representation is then constructed by using certain results of Araki and in the process a new infinitely divisible state on the universal covering group of $SL(2, R)$ is displayed. An ultralocal free theory can also be constructed for the canonical fields, and its relation to the free current theory is shown to involve a certain renormalization procedure reminiscent of the thermodynamic limit.

1. Introduction

In the quantum theory for finite degrees of freedom, it has long been known [1] that all irreducible representations of the quantum mechanical commutation relations, $[p_k, q_j] = -i\delta_{kj}$ $(k, j = 1, 2, \ldots, N)$, are unitarily equivalent[1]. Thus, regardless of the particular Hamiltonian being considered, one may use the standard representation: $p_k \to \frac{1}{i}\frac{d}{dx_k}$, $q_k \to x_k$ on $L^2(R^N, dx)$. In field theory with infinite degrees of freedom, there are many inequivalent irreducible representations of the canonical commutation relations (CCR) [2],

$$[\pi(x), \phi(y)] = -i\delta(x-y) \qquad (x, y \in R^s), \tag{1.1}$$

and it is generally expected that for each Hamiltonian, written in some heuristic fashion in terms of these time-zero fields, one must choose the appropriate representation so that the Hamiltonian can be defined as a bona fide operator on the representation space.

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[1] This result is rigorously true only for the Weyl form of the commutation relations.