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Equivalence of Ensembles in Quantum Lattice Systems: States

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Abstract. The general analysis of the equivalence of ensembles in quantum lattice systems, which was undertaken in paper I of this series, is continued.

The properties of equilibrium states are considered in a variational sense. It is then shown that there exists a canonical as well as a microcanonical variational formulation of equilibrium both of which are equivalent to the grandcanonical formulation.

Equilibrium states are constructed both in the canonical and in the microcanonical formalism by means of suitable limiting procedures.

It is shown, in particular, that the invariant equilibrium states for a given energy and density are those for which the maximum of the mean entropy is reached. The mean entropy thus obtained coincides with the microcanonical entropy.

I. Introduction

In a previous paper [1], the problem of the equivalence of ensembles in Quantum Lattice Systems was begun. The purpose of this paper is to continue the analysis of equivalence of ensembles in quantum spin systems. In [1] we gave an algebraic formulation of the mathematical framework of quantum spin systems in the three usual ensembles and also some equivalence formulas of the respective thermodynamic functions. This allowed us to show some properties in one ensemble if they are proved in another.

In the present we continue in the same way and we consider the properties of the equilibrium states using a variational principle introduced by Ruelle for the grandcanonical ensemble [2].

We consider a quantum lattice system on Z^ν. We associate with each lattice site $x \in Z^\nu$ a Hilbert space $\mathscr{H}_x$ of dimension two, and with each finite region Λ in Z^ν the tensor product

$$\mathscr{H}(\Lambda) = \bigotimes_{x \in \Lambda} \mathscr{H}_x .$$

If $\Lambda_1 \subset \Lambda_2$ we can identify each bounded operator A on $\mathscr{H}(\Lambda_1)$ with $A \otimes 1_{\Lambda_2/\Lambda_1}$ on $\mathscr{H}(\Lambda_2)$, where $1_{\Lambda_2/\Lambda_1}$ is the identity of $\mathscr{H}(\Lambda_2/\Lambda_1)$. With this