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Approximate Vacuums in $(:\phi^{2m}:g(x))_2$ Field Theories and Perturbation Series*

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Abstract. A class of perturbation problems is considered, in which the Rayleigh-Schrödinger perturbation series for the ground state eigenvalue and eigenvector are presumed to diverge. This class includes the $(:\phi^{2m}:g(x))_2$, $(m=2,3)$ quantum field theory models and the quantum mechanical anharmonic oscillator. It is shown that, using matrix elements and vectors which occur in the series coefficients, one may construct convergent approximants to the eigenvalue and eigenvector. Results of a calculation of the ground state energy of the x^4 anharmonic oscillator are given.

I. Introduction

We consider a class of perturbation problems in which the Rayleigh-Schrödinger perturbation series for the ground state eigenvalue and eigenvector of a perturbed operator $H_0+\lambda V$ are presumed to diverge. This class, which is defined in Section II, includes the $(:\phi^{2m}:g(x))_2$, $(m=2,3)$ relativistic quantum field theory models in two-dimensional space-time and the quantum mechanical anharmonic oscillator. In these examples the $R-S$ series are known to diverge [1, 2].

We show that, using matrix elements and vectors which occur in the series coefficients, one may construct convergent approximants to the eigenvalue and eigenvector. Previously, Loeffel *et al.* [3, 2] have shown that the diagonal Padé approximants to the $R-S$ series for the eigenvalues of the x^4 anharmonic oscillator converge and give upper and lower bounds for the eigenvalue $E(\lambda)$, and Simon [4] has shown that the series for the ground state energy of $(:\phi^4:g(x))_2$ is Borel summable to $E(\lambda)$. These results depend on the analytic and asymptotic properties of $E(\lambda)$ for complex λ and do not directly give any information about the eigenvector. Also, the method of Borel summability involves an analytic continuation and hence does not enable one to construct rigorously convergent approximants to $E(\lambda)$. The scheme described in Section III involves solving an approximate eigenvalue problem and gives a monotonically decreasing sequence of upper bounds for $E(\lambda)$ when λ is real

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