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On Factor Representations and the C^*-Algebra of Canonical Commutation Relations

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Abstract. A new C^*-algebra, $\mathscr{A}$, for canonical commutation relations, both in the case of finite and infinite number of degrees of freedom, is defined. It has the property that to each, not necessarily continuous, representation of CCR there corresponds a representation of $\mathscr{A}$. The definition of $\mathscr{A}$ is based on the existence and uniqueness of the factor type II_1 representation. Some continuity properties of separable factor representations are proved.

1. Introduction

In this paper we define and investigate a C^*-algebra for representations of canonical commutation relations (CCR). It will be natural for our considerations to start with a general abelian group $\mathscr{R}$ and a bicharacter b on $\mathscr{R}$ and then define a representation of CCR over $(\mathscr{R}, b)$ as a mapping, say W, from $\mathscr{R}$ to unitary operators on a Hilbert space such that

$$W(x)\, W(y) = b(x, y)\, W(x+y)\,. \tag{1.1}$$

The only condition we impose on b is that it be non-degenerate in a sense given later.

In applications of CCR for the description of quantum systems with infinitely many degrees of freedom one has additional structure, and only representations satisfying certain conditions are of interest. For instance for Bose systems, $\mathscr{R}$ is in fact a vector space, the bicharacter b is defined by a bilinear form and W has to be continuous on rays, i.e. for each $x \in \mathscr{R}$ the one parameter groups $\lambda \mapsto W(\lambda x)$ have to be (weakly) continuous. In statistical mechanics the representations are locally normal with respect to the Fock representations.

For representations continuous on rays, a pertinent C^*-algebra was defined by Segal [11], and the C^*-algebras of statistical mechanics are decribed in [10]. Our algebra is the minimal one. It is contained in every C^*-algebra containing unitary operators satisfying (1.1) and is defined only by $(\mathscr{R}, b)$. The results are completely analogous to those for canonical anticommutation relations [12, 13]. From our point of view this is