Commun. math. Phys. 24, 133—150 (1972)

Asymptotic Behaviour of Spin-Momentum Distribution Observables for Fermion Fields with Cut-Off Self-Coupling*

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Received September 6; in revised form October 7, 1971

Abstract. In a previous paper asymptotic creation and annhilation operators $a^{\#}_{\pm}$ have been constructed by the Kato-Mugibayashi method from the creation and annihilation operators $a^{\#}$ for spin $\frac{1}{2}$ fields with an interaction Hamiltonian density which is an even-degree polynomial in the field with ultra-violet cut-off and its derivatives. For any eigenvector Φ of the total Hamiltonian $H = H_0 + H_I$ partial isometries $\Omega_{\pm}$ have been defined so that $a^{\#}_{\pm}$ equal $\Omega_{\pm}\, a^{\#}\; \Omega^{*}_{\pm}$ on the ranges $\mathscr{F}_{\pm}$ of $\Omega_{\pm}$. Since the existence of a ground-state of H has been proved, the existence of at least one pair $\Omega_{\pm}$ follows. The purpose of this paper is to show that for any $\Psi \in \mathscr{F}_{\pm}$ orthogonal to Φ the distribution of spins and momenta of the interacting Schrödinger states $\exp[-itH]\,\Omega_{\pm}\Psi$ approaches for $t \to \mp\infty$ the distributions of spins and momenta of the free state $\exp[-itH_0]\,\Psi$ if a wave-amplitude renormalization is carried out in $\mathscr{F}_{\pm}$. This is achieved by studying the expectation values of the operators in the *maximally abelian* W^*-algebra $\mathfrak{A}(\tilde{\mathcal{O}})$ generated by operators of the form $\int \varrho a^* a$, in terms of which *any* information about spins and momenta can be expressed.

1. Introduction

The aim of this paper is to demonstrate that the concept of physical wave-operators introduced in Ref. [1] in order to deal with the scattering problem for long-range potentials [2] can be applied also to constructive quantum field theory. This presents the hope for a unified quantum scattering theory formulated in terms of such operators, which would apply to the relativistic as well as non-relativistic case.

The class of models under study [3] are the spin $\frac{1}{2}$ fields $\psi(x)$ which self-interact via an interaction Hamiltonian

$$H_{I,\kappa,v} = \int_{\mathbb{R}^3} v(x)\, H_{I,\kappa}(x)\, d^3 x \tag{1.1}$$

* Supported in part by the National Research Council of Canada.