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Operator Product Expansions and Composite Field Operators in the General Framework of Quantum Field Theory

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Abstract. The short distance behavior of field operator products is analyzed. It is shown that under certain conditions operator product expansions can be derived which give complete information on the short distance behavior and lead to the construction of composite field operators.

1. Introduction

A central problem in local quantum field theory is the definition of products of field operators at the same point. Important examples of such composite field operators are Lagrangian densities, energy-momentum tensors, interaction terms of local field equations and current operators associated with internal symmetries. Let $A_1, \ldots, A_n$ be field operators satisfying the usual postulates of local quantum field theory[1]. The difficulty in constructing composite operators such as

$$A_1(x) \ldots A_a(x) \tag{1.1}$$

originates in the singularity of the operator product

$$A_1(x_1) \ldots A_a(x_a) \tag{1.2}$$

for coinciding arguments. Such singularities inevitably occur as a consequence of relativistic invariance and positive definite metric in Hilbert space [1].

In Ref. [2] an expansion of the operator product (1.2) was proposed which exhibits the singularities near $x_j = x$ and simultaneously allows

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[1] Each of the operators A_j may have several components transforming like a tensor under homogeneous Lorentz transformations.