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On a Limiting Procedure for Obtaining Physical States for an Infinite Fermi System

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Abstract. We propose a limiting procedure for obtaining physical states for an infinite non-relativistic Fermi system. We take the thermodynamic limit of vector states in the Fock representation of the C.A.R. algebra, representing a condensate state of "atoms" each of which is formed by 4 fermions. In a simplified example considered in detail, the limit state has a simple decomposition into the product of two B.C.S. states. If B^+ is the operator creating the "atom" from the vacuum $|\psi_{0F}\rangle$, it is proved that the states obtained by taking the thermodynamic limit of the vector states corresponding to $(B^+)^n|\psi_{0F}\rangle$ and $\sum_{n=0}^{\infty} \mathfrak{z}^{n/2}(B^+)^n/(n!)^2\,|\psi_{0,F}\rangle$ respectively, coincide on the gauge-invariant elements of the algebra for a suitable value of $\mathfrak{z}$.

1. Introduction

We shall present here a procedure for obtaining states over the algebra of the anticommutation relations (C.A.R. algebra) which are locally normal, translation invariant and not quasi-free. We recall that a "quasi-free" state or "generalized free" state is characterized by the property that its truncated (n, m)-point functions $W^T_{n,m}$ vanish if $n+m>2$.

In the simple case considered in this paper, the state of the system belongs to the closed-convex hull of the set of the quasi-free states when the thermodynamic limit is performed. However, the procedure we propose is of a completely general character, and it is probable that new physical states may be found working along the lines of the present work.

Our state is constructed by means of the following procedure[1]:

i) We include the one-dimensional system in a finite box of linear dimension L.

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[1] This procedure has been already indicated in the unpublished report [1]. The idea of including four-particle correlations in a Fermi system is quite old: see, e.g. Flowers [2] and Bremond and Valatin [3]. However, to the best of our knowledge, no workable example of physical state for the infinite system which includes four-particle correlations is known. The state proposed in ref. [3] is a product state of finite dimensional type in Powers terminology [4]. Hence by Theorem 5.20 of Ref. [4], this state becomes quasi-free in the thermodynamic limit if we require translation invariance.