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# More Qualitative Cosmology

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**Abstract.** Standard geometric techniques of differential equation theory are employed to determine the qualitative behaviour of a set of non-rotating perfect-fluid cosmologies, whose spatially homogeneous hypersurfaces admit a 3-parameter group of isometries of Bianchi types I, II, III, V, or VI. In this way we are led to some new exact solutions of the field equations.

The field equations for a broad class of cosmological models are presented in a regularised form, limitations on the use of this procedure are examined, and some suggestions are made of ways of avoiding the difficulties that arise.

## 1. Qualitative Methods

It may be readily shown [5] that, for a certain class of perfect-fluid non-rotating and spatially homogeneous (but usually anisotropic) cosmological models, the Einstein field equations can be written in the form

$$\frac{dx}{d\Omega} = x\left\{(3\gamma-2)(1-x) - \beta'^2 + \frac{2}{3}\Lambda z\right\}, \tag{1.1}$$

$$\frac{dz}{d\Omega} = -2z\left\{1 + \frac{1}{2}(3\gamma-2)x + \frac{1}{2}\beta'^2 - \frac{1}{3}\Lambda z\right\}, \tag{1.2}$$

$$\frac{d\beta_1}{d\Omega} = \beta'_1, \tag{1.3}$$

$$\frac{d\beta_2}{d\Omega} = \beta'_2, \tag{1.4}$$

$$\frac{d\beta'_1}{d\Omega} = \frac{1}{2}\beta'_1\left\{4 - (3\gamma-2)x - \beta'^2 + \frac{2}{3}\Lambda z\right\} - \frac{1}{2}ze^{2\Omega}\frac{\partial V_1}{\partial\beta_1}, \tag{1.5ai}$$

and

$$\frac{d\beta'_2}{d\Omega} = \frac{1}{2}\beta'_2\left\{4 - (3\gamma-2)x - \beta'^2 + \frac{2}{3}\Lambda z\right\} - \frac{1}{2}ze^{2\Omega}\frac{\partial V_1}{\partial\beta_2}, \tag{1.5aii}$$