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The General Stationary Gravitational Vacuum Field of Cylindrical Symmetry

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Abstract. The general stationary vacuum gravitational field of cylindrical symmetry as recently found by Davies and Caplan is even static. The possible Petrov types of the Riemann tensor are I, D or O. In spacelike infinity the spacetime becomes necessarily flat.

1. Introduction

Levy and Robinson [1] have argued that for axisymmetric stationary systems and modulo the vacuum field equations

$$R_{\mu\nu}=0 \tag{1.1}$$

there exists a canonical (cylindrical) coordinate system in which the line element takes the form

$$\begin{aligned} ds^2 = e^{2u}(dt + a\,d\varphi)^2 - e^{2(k-u)}(dr^2 + dz^2) \\ - r^2 e^{-2u} d\varphi^2 \end{aligned} \tag{1.2}$$

$a=0$ corresponds to Weyl's canonical coordinates for the static case. If u, k, a are functions of r only, the line element represents the vacuum gravitational field within or outside an infinite, axially symmetric rotating cylindrical mass distribution and the field Eq. (1.1) reduce to

$$\begin{aligned} \frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} + \frac{1}{2r^2} e^{4u}\left(\frac{da}{dr}\right)^2 = 0 \\ \frac{d^2a}{dr^2} - \frac{1}{r}\frac{da}{dr} + 4\frac{da}{dr}\frac{du}{dr} = 0 \\ \frac{2}{r}\frac{dk}{dr} - 2\left(\frac{du}{dr}\right)^2 + \frac{1}{2r^2} e^{4u}\left(\frac{da}{dr}\right)^2 = 0\,. \end{aligned} \tag{1.3}$$

Recently Davies and Caplan [2] have found the general solution of (1.3), from which they deduced, that under the condition u, a, k to be finite at $r=0$ the interior of the rotating cylinder is flat.