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Isotropic Solutions of the Einstein-Boltzmann Equations

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Abstract. It is shown that in all solutions of the Einstein-Boltzmann equations in which the particle distribution function is isotropic about some 4-velocity field, the distortion of that velocity field vanishes; further, either its expansion or its rotation vanishes. We discuss briefly further kinetic solutions in which the energy-momentum tensor has a perfect fluid form.

1. Introduction

The General-Relativistic theory of a collision-dominated one-component gas is governed by the Einstein gravitational field equations[1]

$$R_{ab} - \tfrac{1}{2} R g_{ab} + \Lambda g_{ab} = T_{ab} \tag{1.1}$$

where the energy-momentum tensor T_{ab} is obtained from a suitable model for the gas. A fluid model is appropriate if the collision dominance is assumed to imply that the gas is sufficiently close to equilibrium to allow the definition of a unique 4-velocity and the use of a conventional thermodynamic formalism. Included in this description is the perfect fluid approximation where one neglects the dissipative effects of heat conduction and shear viscosity.

Though inapplicable at very high densities, within its range of validity relativistic kinetic theory provides a more detailed description than the fluid model. For example it allows one to calculate the form of the transport equations which, in the macroscopic theory, are a phenomenological assumption. To a certain extent kinetic theory may therefore be used to examine the nature and validity of the fluid theory.

In this paper, we study kinetic theory when the one particle distribution function is everywhere isotropic, extending the results of EGS (Ehlers, Geren and Sachs [1]), and then consider the nature of the perfect fluid approximation. Preparatory to stating our precise result, we first briefly review the fluid and kinetic models.

[1] Latin indices run from 0 to 3, Greek indices from 1 to 3. We use square brackets to denote skew symmetrization, round brackets to denote symmetrization. The speed of light is normalized to unity.