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Harmonic Analysis on the Poincaré Group

II. The Fourier Transform

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Abstract. The generalized matrix elements on the Poincaré group are used to write the Fourier transform explicitly. This realizes a mapping between positive type functions on the group and generalized density matrices.

Introduction

This work is the continuation of our preceding paper [1] hereafter referred to as part I. It is written in three chapters, where we deal successively with

I. The matrix elements.

II. The dual space of the Poincaré group.

III. The Fourier transform in $\mathscr{L}^1$ and $\mathscr{L}^2$ spaces.

One gets a fairly simple view of the structure of the dual space of equivalence classes of unitary irreducible representations of the group.

Our aim is to work out the Fourier transform for distributions on the group and we hope this can be achieved in a following and last article.

Chapter I. The Matrix Elements

In part I, we computed the generalized matrix elements of the unitary representations of the Poincaré group. For this purpose, we wrote explicitly the matrix elements of the stabilizator groups, $SU(2)$ and $SU(1, 1)$. We obtained them as eigenfunctions of infinitesimal operators; of course they are only defined up to a phase (this corresponds to the relative (and arbitrary) phase of the vectors of the basis). Here, we use this indetermination to redefine the matrix elements, in order to obtain a better analytic behaviour. We give these definitions in the case of the $SU(1, 1)$ subgroup and obtain the case of $SU(2)$ by analytic continuation.

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