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Almost Positive Perturbations of Positive Selfadjoint Operators

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Abstract. Let A be a positve selfadjoint operator and let B be a symmetric perturbation of A. We establish sufficient conditions for the essential selfadjointness of $A+B$ on domains where A is essentially selfadjoint. The results have application to the $\lambda\phi^4$ field theory in two space-time dimensions.

I. Introduction

Let A be a positive selfadjoint operator with domain $\mathscr{D}(A)$. We establish sufficient conditions for $A+B$ to be essentially selfadjoint on domains where A is essentially selfadjoint, in particular on $\mathscr{C}^\infty(A)=\bigcap\limits_{n=0}^{\infty}\mathscr{D}(A^n)$. The methods used, both generalize and depend crucially upon, two fundamental theorems concerning regular perturbations. We begin, then, by stating these theorems, together with a few definitions. Proofs may be found in [1].

Definition 1.1. An operator A is relatively bounded with respect to an operator T (or T-bounded) if $\mathscr{D}(A)\supset\mathscr{D}(T)$ and if there are constants a and b such that

$$\|A\psi\|^2\leqq a^2\|\psi\|^2+b^2\|T\psi\|^2,\ \psi\in\mathscr{D}(T). \tag{1.1a}$$

The T-bound of A is defined as the greatest lower bound of all non-negative b for which (1.1a) holds.

Definition 1.2. An operator T has *strong control* over an operator A, if A is T-bounded with T-bound strictly less than 1.

Definition 1.3. An operator T has *weak control* over an operator A if A is T bounded and (1.1a) holds with $b=1$.

It is clear that T has weak control over A if it has strong control. It is less clear that A may be T bounded with T bound equal to 1, even though T does not have weak control over A.

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