Commun. math. Phys. 22, 253—258 (1971)

Positivity and Self Adjointness of the $P(\phi)_2$ Hamiltonian

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Received May 15, 1971

Abstract. We give a new proof that the locally correct Hamiltonian $H(g)$ is self adjoint, and that the vacuum energy $E(g) = \inf \text{spectrum} H(g)$ satisfies $-O(D) \leqq E(g)$, where $0 \leqq g \leqq 1$ and $D = \text{diam.supp.} g$.

An existence theorem has been proved for boson quantum fields with polynomial self interactions in two space time dimensions, and many basic properties of these quantum field models have been established. A self contained account of this theory is presented in [1]. A principal step in the construction of the field theory is to show that the Hamiltonian (energy operator) for a bounded space time region is bounded from below and self adjoint. The original proof of semiboundedness was given by Nelson [2] for the ϕ^4 theory. It was extended by Glimm [3] to a different type of space cutoff and to a positive polynomial $P(\phi)$ interaction. The authors [4] obtained a volume independent bound on the vacuum energy per unit volume. The original proof of self adjointness was given by the authors [5] for the ϕ^4 theory and by Rosen [6] for the $P(\phi)$ theory. Subsequent simplifications have been given [7–12]. In this note we present an easy proof of self adjointness and of the volume independent lower bound. The previous simplifications did not yield the volume independent lower bound. See [1] for notation.

Let

$$H(\kappa) = H_{o\,V} + H_I(\kappa) = H_{0,V} + \int :P(\phi_{\kappa,V})(x): g(x)\, dx\,,$$

where g is a function with compact support and $0 \leqq g \leqq 1$. We use the fact that the operators $H(\kappa)$, $H_{o,V}$, $H_I(\kappa)$ are each self adjoint and bounded from below.

* Supported in part by Air Force Office of Scientific Research, Contract AF49(638)–1719.

** On leave at Princeton University. Alfred P. Sloan Foundation Fellow. Supported in part by Air Force Office of Scientific Research, Contract F44620–70–C–0030.