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Invariant Tensors in $SU(3)$

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Abstract. The construction of independent $SU(3)$ tensors out of octets of fields is considered by investigating numerically invariant $SU(3)$ tensors. A method of obtaining independent sets of these to any rank is discussed and also independent sets are explicitly displayed up to fifth rank. It is shown that this approach allows us to obtain relations among the invariant tensors, and useful new identities involving the d_{ijk} and f_{ijk} tensors are exhibited.

I. Introduction

In $SU(3)$ or chiral $SU(3) \otimes SU(3)$ theories of elementary particles, the most common multiplets of particles which we have to handle are the octets or regular representations of $SU(3)$. When dealing with effective Lagrangians [1], especially with the terms in these Lagrangians which break the symmetry, and also when considering non-linear transformations under the chiral group, it is frequently necessary to build $SU(3)$ scalars and tensors out of these basic octets. In practice this means [2] that the quantities of importance are the tensors of $SU(3)/Z(3)$ and henceforth we shall restrict ourselves to this subset of the $SU(3)$ tensors. In practical calculations it is always of the greatest importance to be certain that all possible independent tensors have been written down. In a recent consideration [3] of a model of chiral symmetry breaking, involving three independent octets of fields, we were faced with just such a problem, and it was in the attempt to find a general effective way of dealing with this that the following work arose. This approach allowed us to handle the very complicated expressions which occur in the second and third orders in our calculation with relative ease, and this was not achieved satisfactorily with the usual techniques [2] which we used when we first attempted this problem. Our analysis consists of coupling the octets together with invariant $SU(3)$ tensors and for this purpose we have been able to develop a technique for deriving all these tensors to all orders. With the higher ranks the method is, as could be expected, unwieldy, although nonetheless possible in principle, and probably