Commun. math. Phys. 22, 133—161 (1971)

General Properties of Polymer Systems

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Received February 19, 1971

Abstract. We prove the existence of the thermodynamic limit for the pressure and show that the limit is a convex, continuous function of the chemical potential.

The existence and analyticity properties of the thermodynamic limit for the correlation functions is then derived; we discuss in particular the Mayer Series and the virial expansion.

In the special case of Monomer-Dimer systems it is established that no phase transition is possible; moreover it is shown that the Mayer Series for the density is a series of Stieltjes, which yields upper and lower bounds in terms of Padé approximants.

Finally it is shown that the results obtained for polymer systems can be used to study classical lattice systems.

1. Introduction

In the last decade the mechanism of phase transitions has been intensively investigated by studying properties of rather simple models. One of those models, the Polymer Model [1] (also called quasi-crystalline model), consists of a lattice which is fully covered with non-overlapping "monomers" (molecule occupying one site of the lattice) and "polymers" (rigid system of molecules which can be placed on the lattice in such a way that each molecule of the polymer coincide with one site of the lattice).

This model appears for example in the study of adsorption of polyatomic molecules on a surface; in this case the monomers represent the empty sites. This same model describes physical systems consisting of molecules with unpaired electrons; these molecules then interact to form long chains or polymers. Finally we shall recall that this is also the model which is introduced in the study of liquids consisting of molecules of different sizes or in the so-called cell-cluster theory of liquid state [2].

On the other hand this model is very general since any classical lattice systems (as defined by Ruelle [3]) can be reduced to a system of polymers on the same lattice; this is the "association problem". In some special cases the polymer system associated with the classical lattice

* Work presented in partial fullfilment of the Ph. D. Thesis.