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The Quadratic Lagrangians in General Relativity

V. N. FOLOMESHKIN
Institute of High Energy Physics, Serpukhov, U.S.S.R.

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Abstract. The solutions of the General Relativity equations with quadratic Lagrangians $R_{iklm}R^{iklm}$, $R_{ik}R^{ik}$, R^2 are studied. It is shown that nontrivial Euclidian (at $r\to\infty$) solution of the theory equations does not exist when $T\neq 0$ (T is a trace of the energy-momentum tensor of matter). The Schwarzschild solution is not an external part of a total solution when $T\neq 0$. Under condition $T=R=0$ Lagrangians $R_{iklm}R^{iklm}$, $R_{ik}R^{ik}$ lead to the identical field equations, so there exist the only quadratic Lagrangian and the only field equations. This equation has a solution with an external part being a standard Schwarzschild solution for the statical spherically symmetric case.

1. Introduction

It is known that the standard external Schwarzschild solution satisfies the equations of the quadratic Lagrangians theory. A conclusion is likely to be made that with respect to its experimental consequences the gravitation theory with quadratic Lagrangians

$$L_1 = R_{iklm}R^{iklm}, \quad L_2 = R_{ik}R^{ik}, \quad L_3 = R^2 \tag{1}$$

is equivalent to the usual formulation of General Relativity. However bearing in mind a real distribution of a matter energy-momentum tensor it is not evident that a nontrivial Schwarzschild solution will be the external one for the total spherically symmetric gravitational field described with Lagrangians (1).

It is known that Lagrangian of the Einstein theory $L_0 = R$ is not invariant with respect to a change of the units measuring the interval. The necessity of the such invariance seems to be natural for a zero mass field. So the question arises on the possibility of using the another particular Lagrangian of course while the main theory principles being conserved. The quadratic Lagrangians satisfy all the necessary invariance requirements.

The properties of Lagrangians (1) and ideas of the necessity for the conformal invariance of a gravitation theory were discussed in papers [1–41].