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Representations of Canonical Anticommutation Relations and Implementability of Canonical Transformations

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Abstract. It is proved that irreducible representations of CAR are determined by the groups of implementable automorphisms of the corresponding C^*-algebra. This is done by a study of implementable canonical transformations. Some results in the same directions for factor representations are given.

1. Introduction

Let $\mathfrak{A}$ be a C^*-algebra and let $\mathscr{A}$ be the group of all its automorphisms. $\mathscr{A}$ acts in a natural way in the set of all representations of $\mathfrak{A}$ and for a representation a of $\mathfrak{A}$ let $\mathscr{A}_a$ denote the isotropy subgroup of a, that is, $\mathscr{A}_a$ is the group of all $\tau \in \mathscr{A}$ such that $a \circ \tau$ is equivalent to a.

The mapping $a \mapsto \mathscr{A}_a$ gives a classification of representations. We study here this mapping for irreducible representations of a uniformly hyperfinite (UHF) algebra of Glimm, [2], and prove that in this case it is one-to-one, that is, if $\mathscr{A}_a = \mathscr{A}_b$ then a is equivalent to b.

This is complementary to what is found in [4] by Powers where it is, in particular, proved that $\mathscr{A}$ acts on the set (of the equivalence classes) of irreducible representations of the UHF algebra in a transitive way.

In investigations of physical systems the UHF algebra appears as the C^*-algebra of canonical anticommutation relations (CAR), [7, 5], or as the algebra used for a description of quantum lattice systems. In the case of CAR the C^*-algebra has additional structure, namely, there is given a linear subspace $\mathscr{R}$ which generates $\mathfrak{A}$, which is invariant with respect to involution and on which the norm of $\mathfrak{A}$ is of hilbertian type. The special automorphisms of $\mathfrak{A}$ which leave $\mathscr{R}$ invariant are called canonical, or Bogoliubov, transformations. In this way every canonical transformation gives rise to a unitary operator on $\mathscr{R}$ and conversely: every unitary operator on $\mathscr{R}$ which commutes with involution extends to an automorphism of $\mathfrak{A}$. The group of all canonical transformations is denoted by $\mathscr{K}$ and $\mathscr{K} \cap \mathscr{A}_a$ by $\mathscr{K}_a$.