

Perturbation of Pseudoresolvents and Analyticity in $1/c$ in Relativistic Quantum Mechanics

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Abstract. The analytic functional calculus, relatively bounded and analytic perturbations of pseudoresolvents have been studied. As an application, the nonrelativistic limit of the Dirac and Klein-Gordon operator in the presence of an external static field has been considered. It has been proved that the resolvents of these operators have only a removable singularity at $c = \infty$. This implies the analyticity at $c = \infty$ of the eigenvalues and eigenvectors corresponding to the bound states of the mentioned operators.

In this paper we consider the behaviour of the Dirac and Klein-Gordon operators for a particle in an external static field as functions of the parameter c , where c is the velocity of light, when $c \rightarrow \infty$.

It turns out that the resolvents of the operators $T(c) \pm mc^2$, where $T(c)$ is the relativistic Hamiltonian, are analytic functions of the complex variable c , having only a *removable* singularity at $c = \infty$. The limit for $c \rightarrow \infty$ is a pseudoresolvent connected in a natural way with the resolvent of a Schrödinger operator with the same potential.

We show that the analytic functional calculus, the relatively bounded and analytic perturbation theory for pseudoresolvents hold equally well as in the case of closed operators (see e.g. [1, 3]).

As an application we obtain the following results: The eigenvalues and eigenvectors for the bound states of the relativistic Hamiltonians are analytic functions of c in some neighbourhood of $c = \infty$. For the Dirac operator this is a generalisation of an earlier work of Titchmarsh [6] in which similar results were obtained by other methods for spherically symmetric potentials.

It should be remarked that our results are only local, i.e., there are no estimates of the radii of convergence.

At the end we discuss some difficulties connected with the fact that the scalar products for the Klein-Gordon equation depend on c in a manner which is not analytic at $c = \infty$.