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The Energy Momentum Spectrum and Vacuum Expectation Values in Quantum Field Theory, II

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Abstract. We prove that the $\mathscr{P}(\varphi)_2$ quantum field theory satisfies the spectral condition. The space time translation $a=(x,t)$ is implemented by the unitary group $U(a)=\exp(itH-ixP)$, and the joint spectrum of the energy operator H and the momentum operator P is contained in the forward cone. We also obtain bounds on certain vacuum expectation values of products of field operators. Our proofs involve an analysis of the limit $V\to\infty$ for approximate theories in a periodic box of volume V. Assuming the existence of a uniform mass gap, we are able to establish all the Wightman axioms with the exception of the Lorentz invariance of the vacuum.

1. Introduction

We study a boson quantum field φ with a polynomial self interaction $\mathscr{P}(\varphi)$ in two dimensional space time. This theory provides an example of all the Haag-Kastler axioms and many of the Wightman axioms for quantum field theory. In this paper we prove that the energy-momentum spectrum lies in the forward cone. In addition, we prove bounds on the vacuum expectation values of products of the differentiated field operators $\partial_t\varphi(x,t)=\varphi_t$ and $\partial_x\varphi(x,t)=\varphi_x$. Three of the Wightman axioms remain open problems for the $\mathscr{P}(\varphi)_2$ theory. They are the invariance of the vacuum under Lorentz rotations, the uniqueness of the vacuum and the existence of vacuum expectation values of products of the field $\varphi(x,t)$ (without differentiation). Assuming the existence of a mass gap, we verify the latter two of these three missing axioms in Section 4.

The $\mathscr{P}(\varphi)_2$ theory is obtained as a limit of cutoff or approximate field theories. We have previously considered two different space cutoff

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