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Difficulty with a Kinematic Concept of Unstable Particles: the SZ.-Nagy Extension and the Matthews-Salam-Zwanziger Representation

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Abstract. We discuss the possibility of describing unstable systems, or dissipative systems in general, by vectors in a Hilbert space, evolving in time according to some nonunitary group or semigroup of translations. If the states of the unstable or dissipative system are embedded in a larger Hilbert space containing "decay products" as well, so that the time evolution of the system as a whole becomes unitary, we show that the infinitesimal generator necessarily has all energies from minus to plus infinity in its spectrum. This result supplements and extends the well-known fact that a positive energy spectrum is incompatible with a decay law bounded by a decreasing exponential. As an example of both facts, we discuss Zwanziger's irreducible, nonunitary representation of the Poincaré group; and we find its minimal, unitary extension (the Sz.-Nagy construction). The answer provides a mathematically canonical approach to the Matthews-Salam theory of wave functions for unstable, elementary particles, where the spectrum difficulty was already recognized. We speculate on the possibility that the Matthews-Salam-Zwanziger representation might be a strong coupling approximation in the relativistic version of the Wigner-Weisskopf theory, but we have not shown the existence of a physically acceptable model where that is so.

I. Discussion

There have been some conjectures in recent years that strong interaction resonances in relativistic quantum physics may have a kinematic characterization as "unstable particles," belonging to complex rest mass eigenvalues of a nonunitary representation of the Poincaré group [1–4]. That such resonances might have an intrinsic "integrity"[1] [5] is an appealing idea, suggested by the concept of "nuclear democracy" [6], more particularly by the analytic S-matrix philosophy that resonance

[1] Our use of the word "integrity" is less precise than that of Lurçat [5]. We have in mind the same thing that we mean by the equally fuzzy "kinematic concept of unstable particles." We use it as a term whose potential meaning is to be realized by the success of some mathematical scheme such as that discussed here. Vaguely, it should mean that there are characteristic properties of unstable particles that can be isolated independently of any interaction and discussed without the aid of interaction dependent quantities like the S matrix, or the relation between free and total Hamiltonians.