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A Remark on Yukawa Plus Boson Selfinteraction in Two Space Time Dimensions*

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Abstract. In this note we show how the results of Glimm and Jaffe [5, 6] on the Yukawa quantum field theory in two space time dimensions may be extended to the case where a boson selfinteraction term is added. Local fields are constructed which do not depend on any cut-off and which have the right (anti)-commutation properties for spacelike separated support of the test functions.

1. Introduction

Throughout this paper we shall employ the notations and definitions used in [7] to discuss the Yukawa interaction in two space dimensions. Let

$$H^Y(g,\varkappa) = H_0 + H_I(g,\varkappa) + c(g,\varkappa) \tag{1.1}$$

be the cutoff Hamilton operator describing the Yukawa interaction between a boson field φ and a fermion field ψ. g is a nonnegative space cutoff function and $\varkappa$ a momentum cutoff parameter. $c(g,\varkappa)$ is a renormalization counterterm. Set

$$:P(\varphi)(g): = \sum_{r=0}^{2n} a_r \int :\varphi(x)^{2n}: g(x)\,dx \tag{1.2}$$

where all a_r are real and $a_{2n} > 0$. Define

$$H(g,\varkappa) = H^Y(g,\varkappa) + :P(\varphi)(g): \tag{1.3}$$

Note that $:P(\varphi)(g):$ has only a space cutoff and no momentum cutoff. We shall be considering the family $H(g,\varkappa)$ as $\varkappa \to \infty$. It will be proved that the resolvents of $H(g,\varkappa)$ converge to the resolvent of a selfadjoint Hamilton operator $H(g)$, which locally gives rise to a finite propagation speed.

Further let

$$\psi(f) = \int e^{itH(g)}\,\psi(x)\,e^{-itH(g)}\,f(x,t)\,dx\,dt\,, \tag{1.4a}$$

$$\varphi(f) = \int e^{itH(g)}\,\varphi(x)\,e^{-itH(g)}\,f(x,t)\,dx\,dt\;(f \in C_0^\infty(\mathbb{R}^2)) \tag{1.4b}$$

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